

9 Sample Exam

This sample exam consists of 40 multiple choice questions. It is intended to be completed within 2.5 hours. The answers to the sample exam questions are provided in Appendix C. You are allowed a calculator and access to the tables found in Appendix A.

1. A random sample of the annual salaries of twelve doctors is taken. The following results were obtained:

$$\{ \$127,309, \$210,100, \$215,152, \$122,152, \$131,040, \$137,115, \$162,144, \$366,565, \$138,023, \$244,055, \$251,123, \$259,824 \}.$$

You are given that the average of these twelve salaries is \$180,383.50.

Using the method of moments, find an estimate for α , using the following CDF to model this data: $F(x) = 1 - (100,000/x)^\alpha$, for $x \geq 100,000$.

- (a) 2.044
- (b) 2.144
- (c) 2.244
- (d) 2.344
- (e) 2.444

2. Following Question 1, suppose you have the annual salaries of 4 other doctors: $\{ \$127,500, \$140,200, \$215,000, \$231,500 \}$. You again are interested in fitting the following CDF to this data set: $F(x) = 1 - (100,000/x)^\alpha$, for $x \geq 100,000$. Determine the value of the Kolmogorov-Smirnov test statistic to test the goodness-of-fit at $\alpha = 2$.

- (a) 0.201
- (b) 0.241
- (c) 0.284
- (d) 0.304
- (e) 0.385

3. Four models are fit to a sample of $n = 100$ observations with the following results:

| Model | # of Parameters | Max Loglikelihood |
|-------|-----------------|-------------------|
| I | 1 | -314 |
| II | 2 | -310 |
| III | 3 | -308 |
| IV | 4 | -306 |

Determine the model favored by the Schwarz Bayesian Criterion.

(a) Model I
 (b) Model II
 (c) Model III
 (d) Model IV
 (e) Impossible to determine without more information

4. You are given the following contingency table for categorical data:

| | Group 1 | Group 2 |
|---------|---------|---------|
| Present | 50 | 80 |
| Absent | 40 | 90 |

Find the value of the Chi-Square statistic and whether we can reject the null hypothesis (independent row and column variables) at $\alpha = .10$.

(a) 1.7, and yes, reject H_0
 (b) 1.7, and no, do not reject H_0
 (c) 2.6, and yes, reject H_0
 (d) 2.6, and no, do not reject H_0
 (e) 3.8, and yes, reject H_0

5. Suppose that from a sample of size 16, a 95% confidence interval for μ is (12, 18). Determine \bar{x} and s , respectively.

(a) 12, 15.1
 (b) 15, 22.3
 (c) 15, 5.6
 (d) 31.4, 15
 (e) 15, 18.6

6. A sample of 30 black bears is taken in an attempt to model the head length of black bears. Based on the sample taken, it was determined that $\bar{x} = 45.5\text{cm}$, and $s^2 = 130.4\text{cm}$. From this information, a 98% confidence interval for the true population standard deviation, σ , can be found to be equal to which of the following?

- (a) (8.73, 16.29)
- (b) (8.42, 14.42)
- (c) (7.71, 17.21)
- (d) (6.67, 20.54)
- (e) (11.67, 13.93)

7. A horticultural researcher is investigating the mean height of a certain type of plant. In making inferences about the mean height of this plant, the error of estimation, d can be no larger than 3cm . Given that it is known that $\sigma = 16\text{cm}$, how large of a sample is necessary for the error of estimation to be no larger than 3cm if the horticulturalist requires a 92% confidence on μ ?

- (a) 86
- (b) 87
- (c) 88
- (d) 89
- (e) 96

8. Suppose that from a random sample a 90% confidence interval for the population mean has been found to be $(12.8, 14.3)$. Would $H_0: \mu = 15$ be rejected in favor of $H_1: \mu \neq 15$ at $\alpha = .10$? Choose the best answer.

- (a) definitely yes
- (b) possibly yes
- (c) definitely no
- (d) possibly no
- (e) impossible to tell without more information

9. The F distribution can be sampled from the following:

$$F = \frac{X_1/n_1}{X_2/n_2},$$

where X_1 and X_2 are random variables on n_1 and n_2 degrees of freedom respectively. What probability distribution do X_1 and X_2 follow?

The random variables X_1 and X_2 follow a

- (a) Normal distribution
- (b) T distribution
- (c) χ^2 distribution
- (d) F distribution
- (e) Binomial distribution

10. If $X \sim \chi^2(5)$, determine the 90th percentile of X .

- (a) 1.61
- (b) 9.236
- (c) 11.070
- (d) 12.833
- (e) 16.750

11. A Type I error ensues when

- (a) H_1 is rejected when it is actually false
- (b) H_0 is rejected when it is actually false
- (c) H_1 is rejected when it is actually true
- (d) H_0 is rejected when it is actually true
- (e) none of the above

12. A Type II error ensues when

- (a) a hypothesis test fails to reject H_1 when H_0 is true
- (b) a hypothesis test fails to reject H_1 when H_0 is true
- (c) a hypothesis test fails to reject H_0 when H_1 is false
- (d) a hypothesis test fails to reject H_0 when H_1 is true
- (e) none of the above

13. The *power* of a statistical test is

- (a) $P[\text{not rejecting } H_0 \text{ when } H_1 \text{ is in fact true}]$
- (b) $P[\text{not rejecting } H_0 \text{ when } H_0 \text{ is in fact true}]$
- (c) $P[\text{rejecting } H_0 \text{ when } H_0 \text{ is in fact true}]$
- (d) $P[\text{rejecting } H_0 \text{ when } H_1 \text{ is in fact true}]$
- (e) none of the above

14. Suppose that the annual income for secretaries follows a normal distribution with $\mu = \$48,000$ and $\sigma = \$3,500$. If 30 secretaries are randomly selected, what is the probability that the average income of these 30 secretaries is between \$47,000 and \$49,000?

- (a) 0.6787
- (b) 0.7244
- (c) 0.7601
- (d) 0.8812
- (e) 0.9454

15. Suppose a die is rolled 200 times. Let X represent the average result obtained from all 200 rolls. Determine $Var(X)$.

- (a) .01458
- (b) .25979
- (c) .60212
- (d) .10775
- (e) .12076

16. Suppose a die is rolled 200 times. Then, using the Central Limit Theorem, approximate the probability that the average result from all 200 rolls exceeds 3.7.

- (a) .0485
- (b) .0565
- (c) .0625
- (d) .0775
- (e) .0815

17. Suppose $X \sim N(10, 2)$. Then, according to Chebychev's inequality, $P(|X - 10| \geq 5)$ is

- (a) less than 0.32
- (b) less than 0.28
- (c) less than 0.24
- (d) less than 0.20
- (e) less than 0.16

Use the following information for Questions 18 to 21: You are given the following data set, the length of 28 ants (in mm):

6.1 7.5 7.2 7.5 8.0 8.1 5.2 8.2 7.0 7.0 7.3 6.9 6.2 5.4
5.0 7.9 7.8 6.5 6.4 6.5 6.9 7.1 8.3 6.5 6.2 7.1 7.5 7.6

Assume that this sample of ant lengths is taken from a normal population. You are also given the following:

Pertinent summary statistics: $\bar{x} = 6.96071, s = 0.882479$.

18. What is a 95% confidence interval for μ ?

- (a) (6.6185, 7.3029)
- (b) (6.4820, 7.4394)
- (c) (6.3620, 7.5594)
- (d) (6.2607, 7.6607)
- (e) (6.1063, 7.8151)

19. What is a 95% upper-bound confidence interval for μ ?

- (a) $(-\infty, 7.9890)$
- (b) $(-\infty, 7.8245)$
- (c) $(-\infty, 7.6219)$
- (d) $(-\infty, 7.4912)$
- (e) $(-\infty, 7.2447)$

20. What is a 90% confidence interval for σ ?

- (a) (0.7933, 0.9812)
- (b) (0.7526, 1.1176)
- (c) (0.7240, 1.1410)
- (d) (0.6911, 1.1898)
- (e) (0.6379, 1.2524)

21. What is a 90% upper-bound confidence interval for σ ?

- (a) (0, 0.6127)
- (b) (0, 0.6454)
- (c) (0, 0.9803)
- (d) (0, 1.0774)
- (e) (0, 1.2285)

Use the following information for Questions 22 to 26: Consider the weights of 12 female and 18 male university students, in pounds. The data is as follows:

| Female | | Male | | |
|--------|-----|------|-----|-----|
| 133 | 161 | 175 | 215 | 182 |
| 132 | 144 | 220 | 183 | 178 |
| 156 | 168 | 185 | 245 | 175 |
| 126 | 150 | 165 | 162 | 179 |
| 108 | 161 | 171 | 170 | 163 |
| 132 | 135 | 185 | 166 | 170 |

Female: $\bar{x} = 142.667, s^2 = 309.42424$

Male: $\bar{x} = 182.722, s^2 = 491.15359$

22. Determine a 90% confidence interval for $(\mu_1 - \mu_2)$, the difference between the mean weights of female and male university students.

- (a) $(-52.962, -28.149)$
- (b) $(-54.962, -26.149)$
- (c) $(-56.962, -24.149)$
- (d) $(-58.962, -22.149)$
- (e) $(-60.962, -20.149)$

23. Consider now only the male students. Let us test the hypothesis that $H_0 : \mu = 180$ lbs vs $H_1 : \mu > 180$ lbs at $\alpha = .1$. Performing the hypothesis test using critical values, what is the value of c , the critical value of the test? (*Hint: note that this is “small” sample*).

- (a) 174.2
- (b) 178.9
- (c) 180.3
- (d) 187.0
- (e) 188.3

24. Considering only male students again, and again considering the test $H_0 : \mu = 180$ lbs vs $H_1 : \mu > 180$ lbs at $\alpha = .1$, what would be the resulting p-value?

- (a) between .35 and .30
- (b) between .30 and .25
- (c) between .25 and .20
- (d) between .20 and .15
- (e) between .15 and .10

25. Consider now only the female students. Let us test the hypothesis that $H_0 : \mu = 140$ lbs vs $H_1 : \mu \neq 140$ lbs at $\alpha = .05$. Performing the hypothesis test using critical values, what is the value of t , the test statistic for the test? (*Hint: note again that this is “small” sample*).

- (a) .41668
- (b) .42668
- (c) .43668
- (d) .44668
- (e) .45668

26. Considering only the female students again, and again considering the test $H_0 : \mu = 140$ lbs vs $H_1 : \mu \neq 140$ lbs at $\alpha = .05$, what would be the resulting p-value?

- (a) between .2 and .3
- (b) between .3 and .4
- (c) between .4 and .5
- (d) between .5 and .6
- (e) between .6 and .7

27. Suppose that, in a random sample of 80 people, 12 of these people were smokers and 68 of them were non-smokers. What is a 95% confidence interval for the proportion of the population this sample was drawn from who are non-smokers?

- (a) (0.70, 1.00)
- (b) (0.71, 0.99)
- (c) (0.73, 0.97)
- (d) (0.75, 0.95)
- (e) (0.77, 0.93)

28. Suppose that, in a random sample of 80 people, 12 of these people were smokers and 68 of them were non-smokers. What is an 80% lower-bound confidence interval for the proportion of the population this sample was drawn from who are non-smokers?

- (a) (0.74, ∞)
- (b) (0.76, ∞)
- (c) (0.78, ∞)
- (d) (0.80, ∞)
- (e) (0.82, ∞)

29. Suppose that sampling is from a Weibull population, with CDF $F(x) = 1 - e^{-(\frac{x}{\theta})^\tau}$, $x \geq 0$. Furthermore, suppose that the empirical 20th and 80th percentiles from a sample are 5 and 12 respectively. Determine an estimate for τ using the method of percentile matching.

- (a) 1.7533
- (b) 1.9831
- (c) 2.2569
- (d) 2.6198
- (e) 2.9117

30. Consider the inverse Pareto Distribution, with CDF:

$$F(x) = \left(\frac{x}{x + \theta} \right)^2, \quad x > 0.$$

Supposing that the median from sample data was 32. Determine an estimate for $P(X > 50)$ using the method of percentile matching.

- (a) 0.185
- (b) 0.205
- (c) 0.248
- (d) 0.297
- (e) 0.375

31. Suppose that sampling is from a population following a Normal, $N(\mu, \sigma)$, distribution. We have a sample of values: $\{11, 17, 2, 5, -5\}$. Determine the method of moments estimate for σ .

- (a) 5.81
- (b) 6.45
- (c) 7.12
- (d) 7.54
- (e) 8.43

32. Suppose that sampling is from a population following a single-parameter Pareto distribution, with $f(x) = \alpha x^{-\alpha-1}$, $x > 1$, $\alpha > 0$. We have sample values: $\{13, 7, 12, 5, 2\}$. Determine the maximum likelihood estimate for α .

- (a) 0.138

- (b) 0.238
- (c) 0.338
- (d) 0.438
- (e) 0.538

33. You are given that insurance losses follow an exponential distribution, with CDF $F(x) = 1 - e^{-x/\theta}$, $x > 0$. All you know about these losses is that 10 of them exceeded \$500.00, and 1 of them was less than \$7500.00. Based on this information, determine the maximum likelihood estimate of θ .

- (a) 6,125
- (b) 7,825
- (c) 8,185
- (d) 10,225
- (e) 14,895

34. Consider the estimator $\hat{\theta} = \frac{3}{16}X$ where X is the mean of the continuous probability distribution with density $f(x) = \frac{1}{8}(1+\theta x)$, $-4 < x < 4$. Determine $\text{bias}(\hat{\theta})$.

- (a) 0
- (b) θ
- (c) -4θ
- (d) $\frac{3}{16}\theta$
- (e) Cannot be determined without more information

35. Let X_1, X_2, \dots, X_8 denote a random sample from a population having mean μ and variance μ^2 . Consider the following estimator of μ :

$$\hat{\mu} = \frac{X_1 + X_2 + \dots + X_9}{8}.$$

Determine the $\text{MSE}(\hat{\mu})$.

- (a) $3\mu^2/32$
- (b) $5\mu^2/32$
- (c) $7\mu^2/32$
- (d) $9\mu^2/32$
- (e) $11\mu^2/32$

36. Let X_i be sampled from a continuous uniform distribution over the range $X_i \in [0, \theta]$. Let the estimator for θ be $\hat{\theta} = \max\{X_i\}, i = 1, \dots, n$, the largest order statistic (the largest value in the sample). If the sample size is n , determine $E[\hat{\theta}^2]$.

- (a) $\theta^2/(n + 2)$
- (b) $n/(n + 2)$
- (c) $n\theta^2/(n + 2)$
- (d) $n\theta^2/(n + 4)$
- (e) $(n + 1)\theta^2/n$

37. Suppose that claim sizes are uniformly distributed over the interval $[0, \theta]$. A sample of n claims, denoted by X_1, X_2, \dots, X_n was observed and an estimate of θ was obtained using $\hat{\theta} = \frac{n+1}{n} \max\{X_1, X_2, \dots, X_n\}$. Which of the following statements is true?

- (a) The estimator is biased and consistent
- (b) The estimator is biased and not consistent
- (c) The estimator is unbiased and consistent
- (d) The estimator is unbiased and not consistent
- (e) None of the above

For Questions 38 to 40, use the following information: The following table illustrates the frequency of motorcycle claims for 500 policyholders in one year:

| Number of Claims | Number of Policyholders |
|------------------|-------------------------|
| 0 | 414 |
| 1 | 71 |
| 2 ⁺ | 15 |

38. What is the Chi-Square goodness-of-fit statistic for a postulated Poisson distribution with parameter $\lambda = 0.25$?

- (a) The test statistic is 4.33
- (b) The test statistic is 5.17
- (c) The test statistic is 6.50
- (d) The test statistic is 7.43
- (e) The test statistic is 8.92

39. What is the critical value for the Chi-Square goodness-of-fit test for a postulated Poisson distribution with parameter $\lambda = 0.25$, assuming $\alpha = 5\%$?

- (a) The test statistic is 3.841
- (b) The test statistic is 5.024
- (c) The test statistic is 5.991
- (d) The test statistic is 7.378
- (e) The test statistic is 7.815

40. What is the conclusion for the Chi-Square goodness-of-fit test for a postulated Poisson distribution with parameter $\lambda = 0.25$, assuming $\alpha = 5\%$?

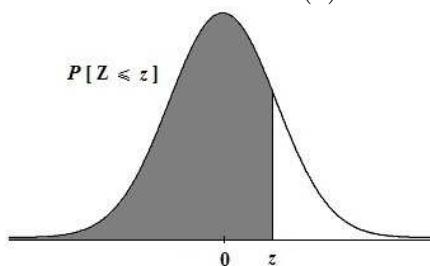
- (a) Reject the null hypothesis that the $\text{Poisson}(\lambda = 0.25)$ distribution is a good fit
- (b) Reject the null hypothesis that the $\text{Poisson}(\lambda = 0.25)$ distribution is a bad fit
- (c) Fail to reject the null hypothesis that the $\text{Poisson}(\lambda = 0.25)$ distribution is a good fit
- (d) Fail to reject the null hypothesis that the $\text{Poisson}(\lambda = 0.25)$ distribution is a bad fit
- (e) None of the above

End of Sample Exam

A Statistical Tables

In this appendix we provide four statistical tables: two for the standard normal distribution, one for the T distribution, and one for the chi-squared distribution.

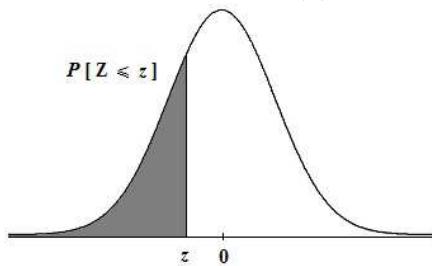
The convention we will employ in this textbook is to choose the value from the table that is nearest to the exact value you seek. The examples in this book, as well as the solutions found in Appendix C, should be consistent with this convention as well.

Standard Normal Distribution: $\Phi(z)$ for Positive z Values

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |

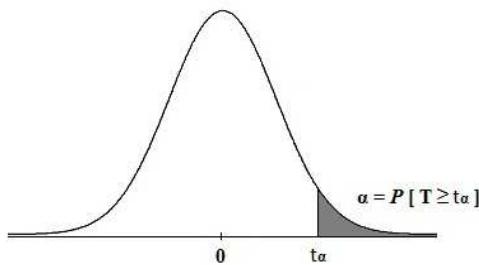
Appendix A. Statistical Tables

Standard Normal Distribution: $\Phi(z)$ for Negative z Values



| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| -0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |
| -0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| -0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| -0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| -0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| -0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| -0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| -0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| -0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| -0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| -1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| -1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| -1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| -1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| -1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| -1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| -1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| -1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| -1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| -1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| -2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| -2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| -2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| -2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| -2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| -2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| -2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| -2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| -2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| -2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| -3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| -3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| -3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| -3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| -3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| -3.5 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
| -3.6 | 0.0002 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |

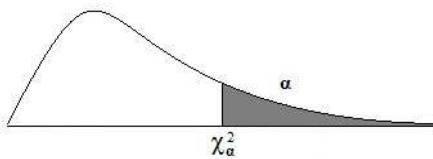
T Distribution: Tail Probability



| d.f. \ α | 0.350 | 0.300 | 0.250 | 0.200 | 0.150 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
|-----------------|-------|-------|-------|-------|-------|-------|-------|--------|--------|--------|
| 1 | 0.510 | 0.727 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 0.445 | 0.617 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 0.424 | 0.584 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 0.414 | 0.569 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 0.408 | 0.559 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 0.404 | 0.553 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 0.402 | 0.549 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 0.399 | 0.546 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 0.398 | 0.543 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 0.397 | 0.542 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 0.396 | 0.540 | 0.697 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 0.395 | 0.539 | 0.695 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 0.394 | 0.538 | 0.694 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 0.393 | 0.537 | 0.692 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 0.393 | 0.536 | 0.691 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 0.392 | 0.535 | 0.690 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 0.392 | 0.534 | 0.689 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 0.392 | 0.534 | 0.688 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 0.391 | 0.533 | 0.688 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 0.391 | 0.533 | 0.687 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 0.391 | 0.532 | 0.686 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 0.390 | 0.532 | 0.686 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 0.390 | 0.532 | 0.685 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 0.390 | 0.531 | 0.685 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 0.390 | 0.531 | 0.684 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 0.390 | 0.531 | 0.684 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 0.389 | 0.531 | 0.684 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 0.389 | 0.530 | 0.683 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 0.389 | 0.530 | 0.683 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 0.389 | 0.530 | 0.683 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 40 | 0.388 | 0.529 | 0.681 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 50 | 0.388 | 0.528 | 0.679 | 0.849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 |
| 60 | 0.387 | 0.527 | 0.679 | 0.848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 70 | 0.387 | 0.527 | 0.678 | 0.847 | 1.044 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 |
| 80 | 0.387 | 0.526 | 0.678 | 0.846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 |
| 115 | 0.386 | 0.526 | 0.677 | 0.845 | 1.041 | 1.289 | 1.658 | 1.981 | 2.359 | 2.619 |
| ∞ | 0.385 | 0.524 | 0.674 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

Appendix A. Statistical Tables

χ^2 Distribution: Right Tail Probability



| d.f. \ α | 0.990 | 0.975 | 0.950 | 0.900 | 0.500 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
|-----------------|--------|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| 1 | 0.000 | 0.001 | 0.004 | 0.016 | 0.455 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.020 | 0.051 | 0.103 | 0.211 | 1.386 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.115 | 0.216 | 0.352 | 0.584 | 2.366 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.297 | 0.484 | 0.711 | 1.064 | 3.357 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.554 | 0.831 | 1.145 | 1.610 | 4.351 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 |
| 6 | 0.872 | 1.237 | 1.635 | 2.204 | 5.348 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 1.239 | 1.690 | 2.167 | 2.833 | 6.346 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.646 | 2.180 | 2.733 | 3.490 | 7.344 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 2.088 | 2.700 | 3.325 | 4.168 | 8.343 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.558 | 3.247 | 3.940 | 4.865 | 9.342 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 3.053 | 3.816 | 4.575 | 5.578 | 10.341 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.571 | 4.404 | 5.226 | 6.304 | 11.340 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 4.107 | 5.009 | 5.892 | 7.042 | 12.340 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.660 | 5.629 | 6.571 | 7.790 | 13.339 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 5.229 | 6.262 | 7.261 | 8.547 | 14.339 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.812 | 6.908 | 7.962 | 9.312 | 15.338 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 6.408 | 7.564 | 8.672 | 10.085 | 16.338 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 7.015 | 8.231 | 9.390 | 10.865 | 17.338 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 7.633 | 8.907 | 10.117 | 11.651 | 18.338 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 8.260 | 9.591 | 10.851 | 12.443 | 19.337 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.897 | 10.283 | 11.591 | 13.240 | 20.337 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 9.542 | 10.982 | 12.338 | 14.041 | 21.337 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 10.196 | 11.689 | 13.091 | 14.848 | 22.337 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 10.856 | 12.401 | 13.848 | 15.659 | 23.337 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 |
| 25 | 11.524 | 13.120 | 14.611 | 16.473 | 24.337 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 12.198 | 13.844 | 15.379 | 17.292 | 25.336 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 12.879 | 14.573 | 16.151 | 18.114 | 26.336 | 36.741 | 40.113 | 43.195 | 46.963 | 49.645 |
| 28 | 13.565 | 15.308 | 16.928 | 18.939 | 27.336 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 14.256 | 16.047 | 17.708 | 19.768 | 28.336 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 |
| 30 | 14.953 | 16.791 | 18.493 | 20.599 | 29.336 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 35 | 18.509 | 20.569 | 22.465 | 24.797 | 34.336 | 46.059 | 49.802 | 53.203 | 57.342 | 60.275 |
| 40 | 22.164 | 24.433 | 26.509 | 29.051 | 39.335 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| 50 | 29.707 | 32.357 | 34.764 | 37.689 | 49.335 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| 60 | 37.485 | 40.482 | 43.188 | 46.459 | 59.335 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| 70 | 45.442 | 48.758 | 51.739 | 55.329 | 69.334 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| 80 | 53.540 | 57.153 | 60.391 | 64.278 | 79.334 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| 90 | 61.754 | 65.647 | 69.126 | 73.291 | 89.334 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |

Appendix C. Solutions to Exercises

4. For an unbiased estimator, the mean squared error is always equal to the variance.
5. One computational advantage of using mean squared error is that it is not a function of the true value of the parameter.

Answer: (4) - For an unbiased estimator, the mean squared error is always equal to the variance.

Sample Exam

1. (c)
2. (e)
3. (b)
4. (b)
5. (c)
6. (a)
7. (b)
8. (a)
9. (c)
10. (b)
11. (d)
12. (d)
13. (d)
14. (d)
15. (a)
16. (a)
17. (e)
18. (a)
19. (e)
20. (c)
21. (d)
22. (a)
23. (d)
24. (a)
25. (b)
26. (e)
27. (e)

- 28. (e)
- 29. (c)
- 30. (e)
- 31. (d)
- 32. (e)
- 33. (c)
- 34. (a)
- 35. (b)
- 36. (c)
- 37. (c)
- 38. (e)
- 39. (a)
- 40. (a)